A RELATIONS AND FUNCTIONS

The charges for parking a car in a short-term car park at an Airport are given in the table shown alongside.

Car park charges	
Period (h)	*Charge*
0 - 1 hours	\$5.00
1 - 2 hours	\$9.00
2 - 3 hours	\$11.00
3 - 6 hours	\$13.00
6 - 9 hours	\$18.00
9 - 12 hours	\$22.00
12 - 24 hours	\$28.00

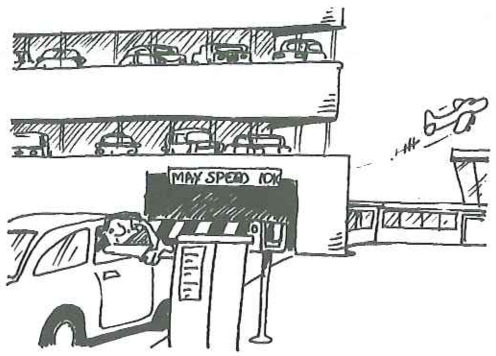
There is an obvious relationship between time spent and the cost. The cost is dependent on the length of time the car is parked.

Looking at this table we might ask: How much would be charged for exactly one hour? Would it be \$5 or \$9?

To make the situation clear, and to avoid confusion, we could adjust the table and draw a graph. We need to indicate that 2-3 hours really means for time over 2 hours up to and including 3 hours i.e., $2 < t \leqslant 3$.

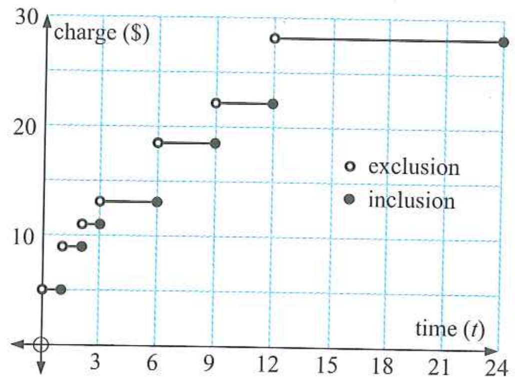
So, we now have

Car park charges	
Period	*Charge*
$0 < t \leqslant 1$ hours	\$5.00
$1 < t \leqslant 2$ hours	\$9.00
$2 < t \leqslant 3$ hours	\$11.00
$3 < t \leqslant 6$ hours	\$13.00
$6 < t \leqslant 9$ hours	\$18.00
$9 < t \leqslant 12$ hours	\$22.00
$12 < t \leqslant 24$ hours	\$28.00



In mathematical terms, because we have a relationship between two variables, time and cost, the schedule of charges is an example of a **relation**.

A relation may consist of a finite number of ordered pairs, such as $\{(1, 5),\ (-2, 3),\ (4, 3),\ (1, 6)\}$ or an infinite number of ordered pairs.



The parking charges example is clearly the latter as any real value of time (t hours) in the interval $0 < t \leqslant 24$ is represented.

The set of possible values of the variable on the horizontal axis is called the **domain** of the relation.

For example:
- $\{t: \ 0 < t \leqslant 24\}$ is the domain for the car park relation
- $\{-2, 1, 4\}$ is the domain of $\{(1, 5), (-2, 3), (4, 3), (1, 6)\}$.

The set which describes the possible y-values is called the **range** of the relation.

For example:
- the range of the car park relation is $\{5, 9, 11, 13, 18, 22, 28\}$
- the range of $\{(1, 5), (-2, 3), (4, 3), (1, 6)\}$ is $\{3, 5, 6\}$.

We will now look at relations and functions more formally.

RELATIONS

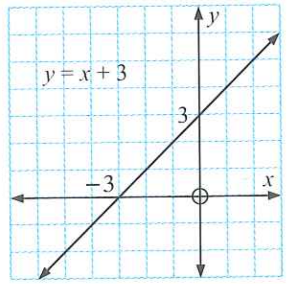
A **relation** is any set of points on the Cartesian plane.

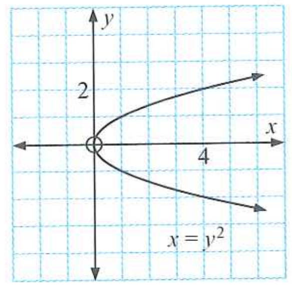
A relation is often expressed in the form of an **equation** connecting the **variables** x and y.

For example $y = x + 3$ and $x = y^2$ are the equations of two relations.

These equations generate sets of ordered pairs.

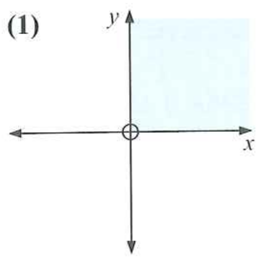
Their graphs are:





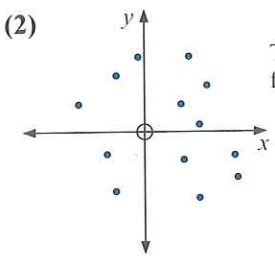
However, a relation may not be able to be defined by an equation. Below are two examples which show this:

(1)



All points in the first quadrant are a relation. $x > 0, y > 0$

(2)



These 13 points form a relation.

FUNCTIONS

A **function** is a relation in which no two different ordered pairs have the same x-coordinate (first member).

We can see from the above definition that a function is a special type of relation.

TESTING FOR FUNCTIONS

Algebraic Test:

If a relation is given as an equation, and the substitution of any value for x results in one and only one value of y, we have a function.

For example:
- $y = 3x - 1$ is a function, as for any value of x there is only one value of y
- $x = y^2$ is not a function since if $x = 4$, say, then $y = \pm 2$.

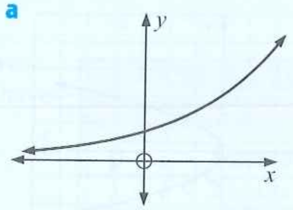
Geometric Test ("Vertical Line Test"):

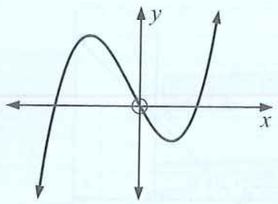
If we draw all possible vertical lines on the graph of a relation, the relation:
- is a function if each line cuts the graph no more than once
- is not a function if one line cuts the graph more than once.

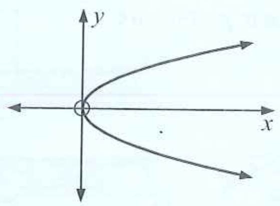
DEMO

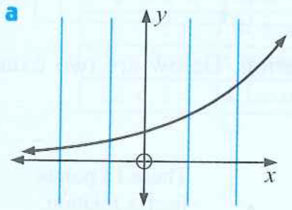
Example 1

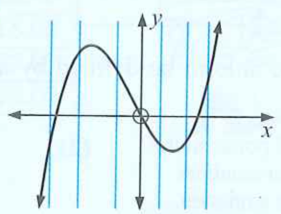
Which of the following relations are functions?

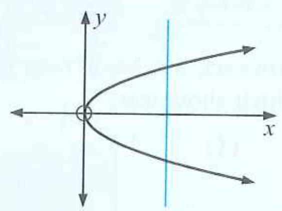
a


b


c


a

a function

b

a function

c

not a function

GRAPHICAL NOTE

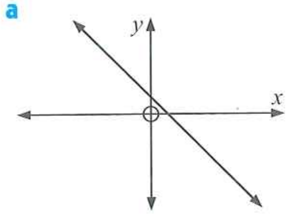
- If a graph contains a small **open circle** end point such as , the end point is **not included**.
- If a graph contains a small **filled-in circle** end point such as , the end point **is included**.
- If a graph contains an **arrow head** at an end such as then the graph continues indefinitely in that general direction, or the shape may repeat as it has done previously.

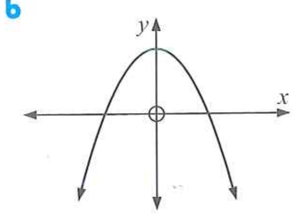
EXERCISE 9A

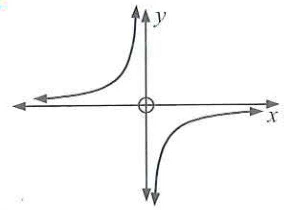
1 Which of the following sets of ordered pairs are functions? Give reasons.

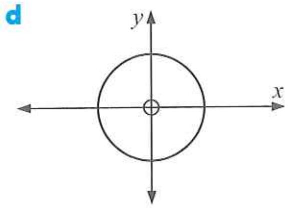
a (1, 3), (2, 4), (3, 5), (4, 6)
b (1, 3), (3, 2), (1, 7), (−1, 4)
c (2, −1), (2, 0), (2, 3), (2, 11)
d (7, 6), (5, 6), (3, 6), (−4, 6)
e (0, 0), (1, 0), (3, 0), (5, 0)
f (0, 0), (0, −2), (0, 2), (0, 4)

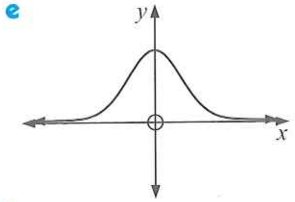
2 Use the vertical line test to determine which of the following relations are functions:

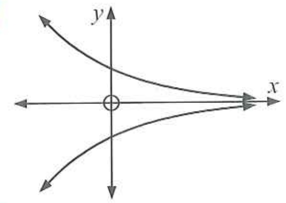
a


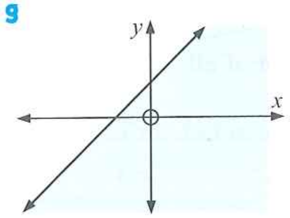
b


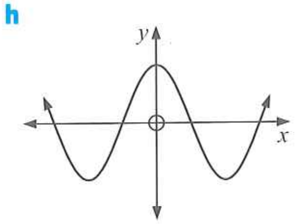
c


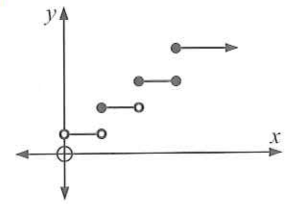
d


e


f


g


h


i


3 Will the graph of a straight line always be a function? Give evidence.

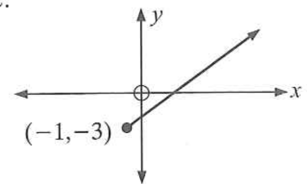
4 Give algebraic evidence to show that the relation $x^2 + y^2 = 9$ is not a function.

B INTERVAL NOTATION, DOMAIN AND RANGE

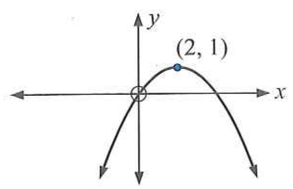
DOMAIN AND RANGE

The **domain** of a relation is the set of permissible values that x may have.
The **range** of a relation is the set of permissible values that y may have.

For example:

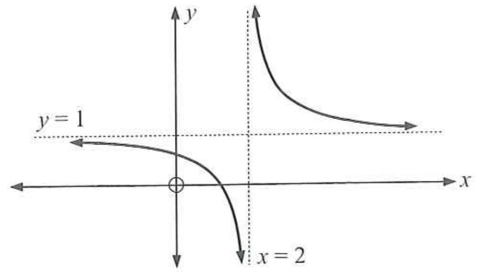
(1)


All values of $x \geqslant -1$ are permissible.
So, the domain is $\{x\colon\ x \geqslant -1\}$.
All values of $y \geqslant -3$ are permissible.
So, the range is $\{y\colon\ y \geqslant -3\}$.

(2)


x can take any value.
So, the domain is $\{x\colon\ x \text{ is in } R\}$.
y cannot be > 1
$\therefore$ range is $\{y\colon\ y \leqslant 1\}$.

(3)



x can take all values except $x = 2$.

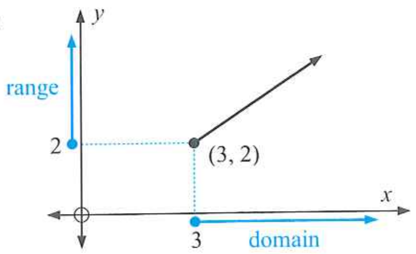
So, the domain is $\{x\colon\ x \neq 2\}$.

Likewise, the range is $\{y\colon\ y \neq 1\}$.

Note: R represents the set of all real values, i.e., all numbers on the number line.

The domain and range of a relation are best described where appropriate using **interval notation**.

For example:

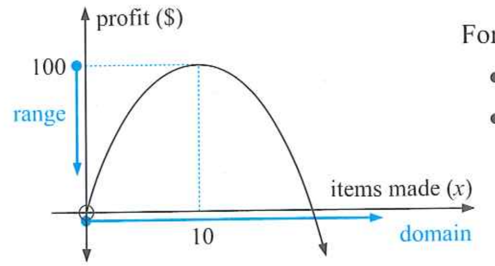


The domain consists of all real x such that $x \geqslant 3$ and we write this as

$\{x :\ x \geqslant 3\}$.

the set of all such that

Likewise the range would be $\{y\colon\ y \geqslant 2\}$.



For this profit function:

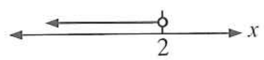
- the domain is $\{x\colon\ x \geqslant 0\}$
- the range is $\{y\colon\ y \leqslant 100\}$.

Intervals have corresponding graphs.

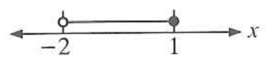
For example:

$\{x\colon\ x \geqslant 3\}$ is read "the set of all x such that x is greater than or equal to 3" and has number line graph

$\{x\colon\ x < 2\}$ has number line graph



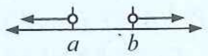
$\{x\colon\ -2 < x \leqslant 1\}$ has number line graph



$\{x\colon\ x \leqslant 0\ \text{ or }\ x > 4\}$ has number line graph

Note:

for numbers *between* a and b we write $a < x < b$

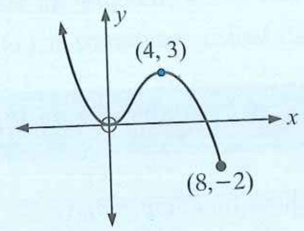


for numbers '*outside*' a and b we write $x < a$ or $x > b$

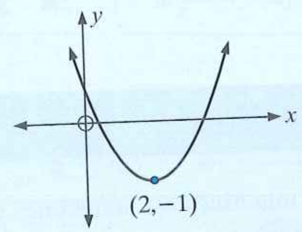
Example 2

For each of the following graphs state the domain and range:

a



b



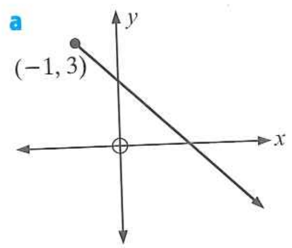
a Domain is $\{x\colon x \leqslant 8\}$.
Range is $\{y\colon\ y \geqslant -2\}$.

b Domain is $\{x\colon\ x \text{ is in } R\}$.
Range is $\{y\colon\ y \geqslant -1\}$.

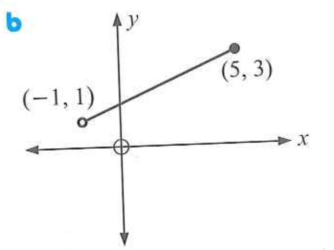
EXERCISE 9B

1 For each of the following graphs find the domain and range:

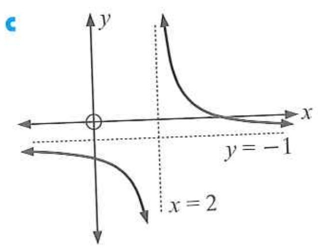
a



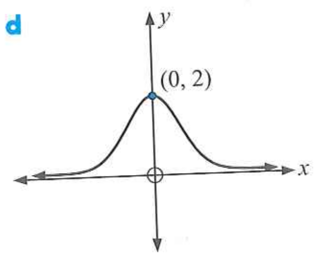
b



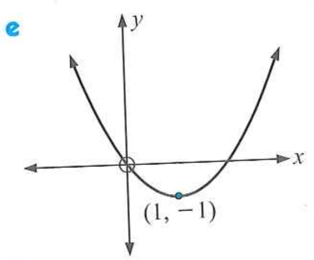
c



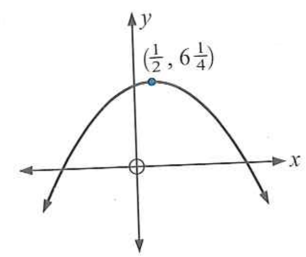
d



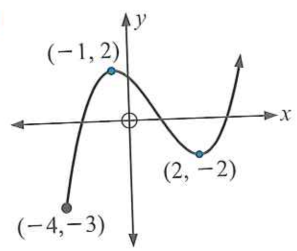
e



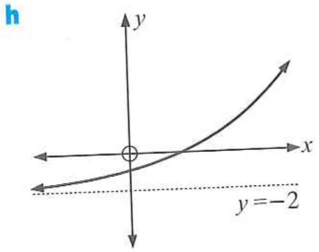
f



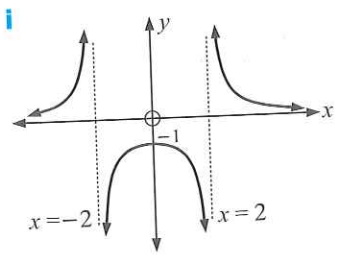
g



h



i



2 Use a graphics calculator to help sketch carefully the graphs of the following functions and find the domain and range of each:

a $y = \sqrt{x}$ **b** $y = \dfrac{1}{x^2}$ **c** $y = \sqrt{4 - x}$

d $y = x^2 - 7x + 10$ **e** $y = 5x - 3x^2$ **f** $y = x + \dfrac{1}{x}$

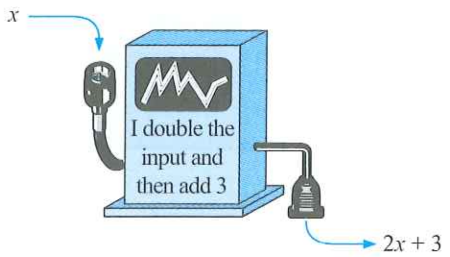
g $y = \dfrac{x+4}{x-2}$ **h** $y = x^3 - 3x^2 - 9x + 10$ **i** $y = \dfrac{3x-9}{x^2-x-2}$

j $y = x^2 + x^{-2}$ **k** $y = x^3 + \dfrac{1}{x^3}$ **l** $y = x^4 + 4x^3 - 16x + 3$

C FUNCTION NOTATION

Function machines are sometimes used to illustrate how functions behave.

For example:



So, if 4 is fed into the machine, $2(4) + 3 = 11$ comes out.

The above 'machine' has been programmed to perform a particular function.
If f is used to represent that particular function we can write:

f is the function that will convert x into $2x + 3$.

So, f would convert 2 into $2(2) + 3 = 7$ and
-4 into $2(-4) + 3 = -5$.

This function can be written as:

$$\boldsymbol{f : x \longmapsto 2x + 3}$$

function f such that x is converted into $2x + 3$

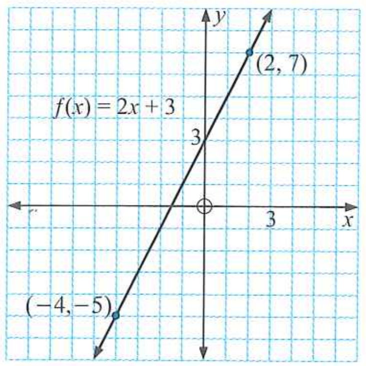
Two other equivalent forms we use are: $f(x) = 2x + 3$ or $y = 2x + 3$

So, $f(x)$ is the value of y for a given value of x, i.e., $y = f(x)$.

Notice that for $f(x) = 2x + 3$, $f(2) = 2(2) + 3 = 7$ and
$f(-4) = 2(-4) + 3 = -5$.

Consequently, $f(2) = 7$ indicates that the point $(2, 7)$ lies on the graph of the function.

Likewise $f(-4) = -5$ indicates that the point $(-4, -5)$ also lies on the graph.



Note:
- $f(x)$ is read as "f of x" and is the value of the function at any value of x.
- If (x, y) is any point on the graph then $y = f(x)$.
- f is the function which converts x into $f(x)$, i.e., $f : x \longmapsto f(x)$.
- $f(x)$ is sometimes called the **image** of x.

Example 3

If $f : x \longmapsto 2x^2 - 3x$, find the value of: **a** $f(5)$ **b** $f(-4)$

$f(x) = 2x^2 - 3x$

a $f(5) = 2(5)^2 - 3(5)$ {replacing x by (5)}
$= 2 \times 25 - 15$
$= 35$

b $f(-4) = 2(-4)^2 - 3(-4)$ {replacing x by (-4)}
$= 2(16) + 12$
$= 44$

The **table mode** on your calculator can be used to find $f(x)$ given x. Click on the icon for instructions.

EXERCISE 9C

1 If $f : x \longmapsto 3x + 2$, find the value of:
a $f(0)$ **b** $f(2)$ **c** $f(-1)$ **d** $f(-5)$ **e** $f(-\frac{1}{3})$

2 If $g : x \longmapsto x - \dfrac{4}{x}$, find the value of:
a $g(1)$ **b** $g(4)$ **c** $g(-1)$ **d** $g(-4)$ **e** $g(-\frac{1}{2})$

3 If $f : x \longmapsto 3x - x^2 + 2$, use the **table mode** of your calculator to find:
a $f(0)$ **b** $f(3)$ **c** $f(-3)$ **d** $f(-7)$ **e** $f(\frac{3}{2})$

Example 4

If $f(x) = 5 - x - x^2$, find in simplest form: **a** $f(-x)$ **b** $f(x+2)$

a $f(-x) = 5 - (-x) - (-x)^2$ {replacing x by $(-x)$}
$= 5 + x - x^2$

b $f(x+2) = 5 - (x+2) - (x+2)^2$ {replacing x by $(x+2)$}
$= 5 - x - 2 - [x^2 + 4x + 4]$
$= 3 - x - x^2 - 4x - 4$
$= -x^2 - 5x - 1$

4 If $f(x) = 7 - 3x$, find in simplest form:
a $f(a)$ **b** $f(-a)$ **c** $f(a+3)$ **d** $f(b-1)$ **e** $f(x+2)$

5 If $F(x) = 2x^2 + 3x - 1$, find in simplest form:
a $F(x+4)$ **b** $F(2-x)$ **c** $F(-x)$ **d** $F(x^2)$ **e** $F(x^2-1)$

6 If $G(x) = \dfrac{2x+3}{x-4}$:
a evaluate **i** $G(2)$ **ii** $G(0)$ **iii** $G(-\frac{1}{2})$
b find a value of x where $G(x)$ does not exist
c find $G(x+2)$ in simplest form
d find x if $G(x) = -3$.

7 f represents a function. What is the difference in meaning between f and $f(x)$?

8 If $f(x) = 2^x$, show that $f(a)f(b) = f(a+b)$.

9 Given $f(x) = x^2$ find in simplest form: **a** $\dfrac{f(x)-f(3)}{x-3}$ **b** $\dfrac{f(2+h)-f(2)}{h}$

10 If the value of a photocopier t years after purchase is given by $V(t) = 9650 - 860t$ dollars:
a find $V(4)$ and state what $V(4)$ means
b find t when $V(t) = 5780$ and explain what this represents
c find the original purchase price of the photocopier.

11 On the same set of axes draw the graphs of three different functions $f(x)$ such that $f(2) = 1$ and $f(5) = 3$.

12 Find $f(x) = ax + b$, a linear function, in which $f(2) = 1$ and $f(-3) = 11$.

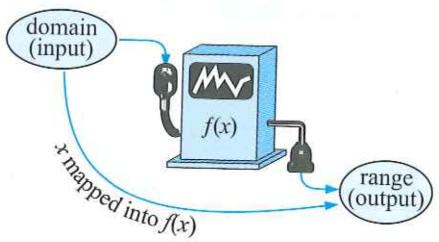
13 Find constants a and b where $f(x) = ax + \dfrac{b}{x}$ and $f(1) = 1$, $f(2) = 5$.

14 Given $T(x) = ax^2 + bx + c$, find a, b and c if $T(0) = -4$, $T(1) = -2$ and $T(2) = 6$.

D FUNCTIONS AS MAPPINGS

In the previous section, we introduced functions as 'machines' which convert x values into $f(x)$ values.

Hence, $f\colon x \longmapsto f(x)$ can be thought of as a number crunching machine which **maps** elements in the domain (input) to elements in the range (output).

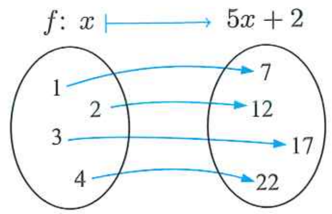


For example, if $f\colon x \longmapsto 5x+2$, we say this as "$f$ is a function which maps x values into two more than five lots of the x values".

If we specify a domain (i.e., we limit the input possibilities) as $\{x\colon x \in Z,\ 1 \leqslant x \leqslant 4\}$, then the permissible values of $f(x)$ are restricted to:

$$\begin{aligned} f(1) &= 5(1)+2 = 7 \\ f(2) &= 5(2)+2 = 12 \\ f(3) &= 5(3)+2 = 17 \\ f(4) &= 5(4)+2 = 22 \end{aligned}$$

Hence, we can represent $f\colon x \longmapsto 5x+2$ on the domain specified using a mapping diagram:



Notice that each element of the domain $\{1, 2, 3, 4\}$ corresponds to a single element within the range $\{7, 12, 17, 22\}$.

For function mappings (as opposed to relation mappings):

- It is possible that multiple elements in the domain correspond to the **same** element within the range.
- It is not possible for a single element in the domain to have more than one corresponding element within the range.

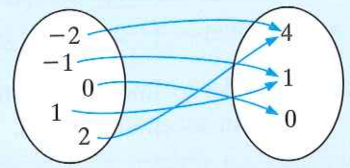
Example 5

Consider $f : x \longmapsto x^2$ for the domain $\{-2, -1, 0, 1, 2\}$

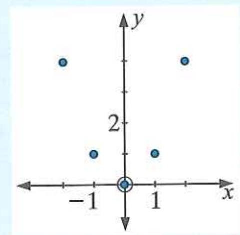
x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

$f(-2) = f(2) = 4$
$f(-1) = f(1) = 1$

Hence, we have a resultant mapping diagram showing some elements within the domain mapping to the same element within the range.



We can also represent this on a set of axes.

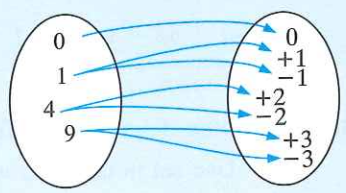


Note: We do not join the elements using a straight line.

Example 6

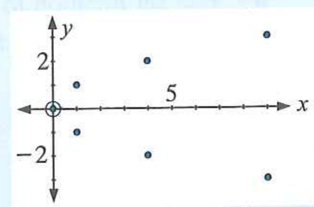
Consider: $p : x \longmapsto \pm\sqrt{x}$ for the domain $\{0, 1, 4, 9\}$

$$\begin{aligned} p(0) &= 0 \\ p(1) &= \pm 1 \\ p(4) &= \pm 2 \\ p(9) &= \pm 3 \end{aligned}$$



Hence out mapping diagram shows single elements of the domain mapping to **multiple** elements within the range. Hence, this is a relation mapping.

If we represent this function on a set of axes it is clear p is not a function.



EXERCISE 9D

1 For the following functions of $f : x \mapsto f(x)$ on $-2 \leqslant x \leqslant 2$ where $x \in Z$:

- **i** draw a mapping diagram to represent $f(x)$
- **ii** list the elements of the domain of $f(x)$ using set notation
- **iii** list the elements of the range of $f(x)$ using set notation.

a $f(x) = 3x - 1$ **b** $f(x) = x^2 + 1$ **c** $f(x) = 3 - 4x$

d $f(x) = 2x^2 - x + 1$ **e** $f(x) = x^3$ **f** $f(x) = 3^x$

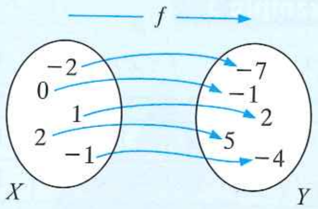
g $f(x) = \dfrac{1}{x+3}$ **h** $f(x) = \dfrac{x+3}{x}$; $x \neq 0$

Example 7

The diagram (right) shows a function f mapping members of a set X to members of set Y.

a Using set notation, write down the members of the domain and range.

b Find the equation of the function f.



a Domain = $\{-2, -1, 0, 1, 2\}$ Range = $\{-7, -4, -1, 2, 5\}$

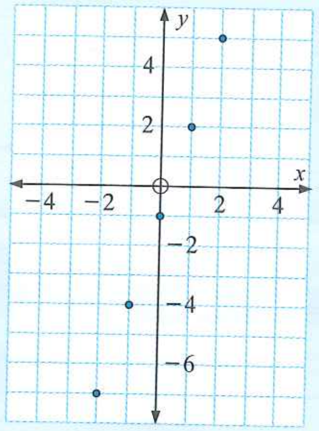
b In order to find the equation $f(x)$, it is very useful to use a table of values (put in ascending order),

i.e.,

x	-2	-1	0	1	2
$f(2)$	-7	-45	-1	2	5

A scatterplot also helps to determine the type of function (i.e., linear, quadratic, etc).

It is obvious from this scatterplot that $f(x)$ is linear.



$\therefore \quad f(x) = mx + c$

where $m = \dfrac{5-2}{2-1}$ {using (2, 5) and (1, 2)}

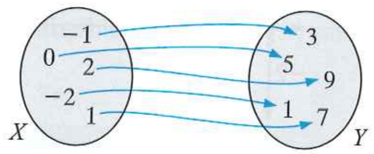
$\therefore \quad m = 3$

$\therefore \quad f(x) = 3x - 1$ or $f : x \mapsto 3x - 1$

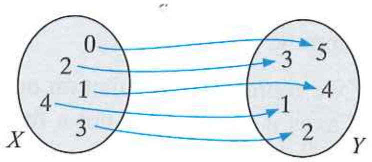
2 For these mapping diagrams, the function f maps elements of X to elements of Y.

- **i** Use set notation to write down the domain of f.
- **ii** Use set notation to write down the range of f.
- **iii** Find the equation of the function f.

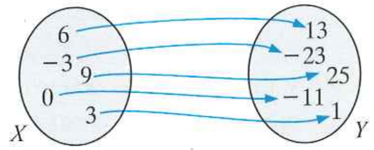
a



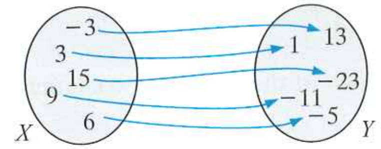
b



c



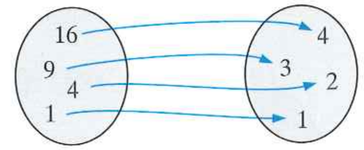
d



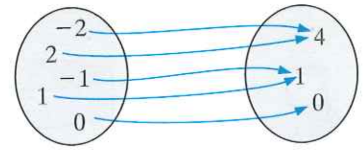
3 For the following mapping diagrams, where f maps X to Y:

- **i** sketch the domain (x) and range (y) on a set of axes
- **ii** find the equation of the function f.

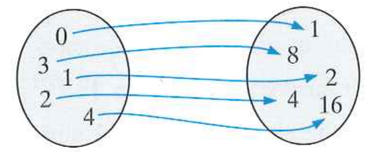
a



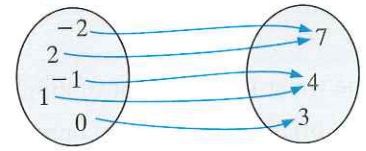
b



c

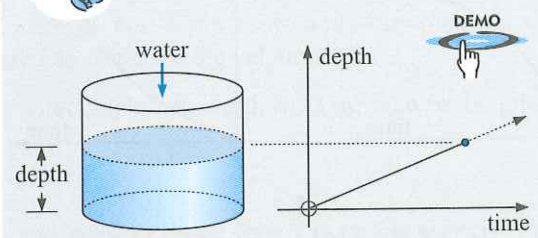


d



INVESTIGATION 1 FLUID FILLING FUNCTIONS

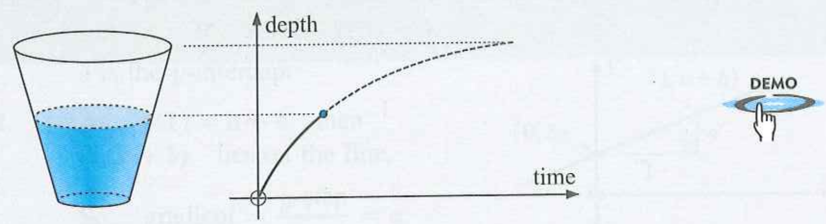
When water is added at a **constant rate** to a cylindrical container the depth of water in the container is a function of time.



This is because the volume of water added is directly proportional to the time taken to add it. If water was not added at a constant rate the direct proportionality would not exist.

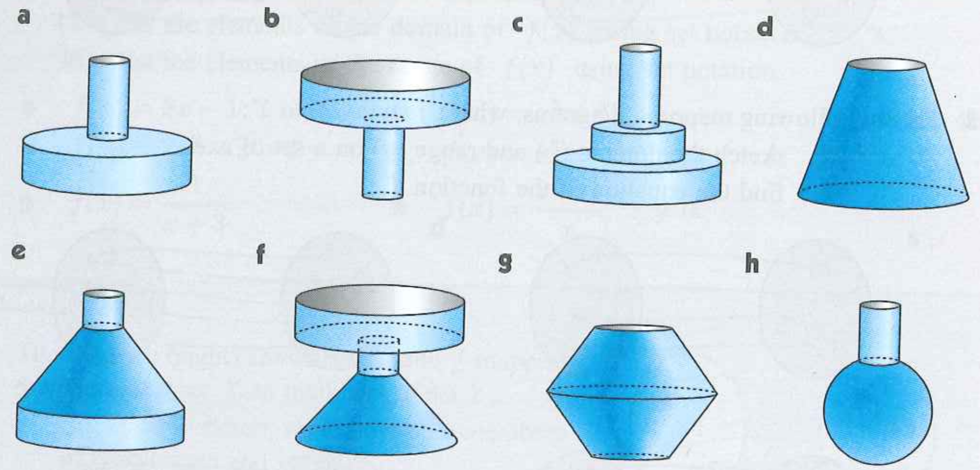
The depth-time graph for the case of a cylinder would be as shown alongside.

The question arises: 'What changes in appearance of the graph occur for different shaped containers?' Consider a vase of conical shape.



What to do:

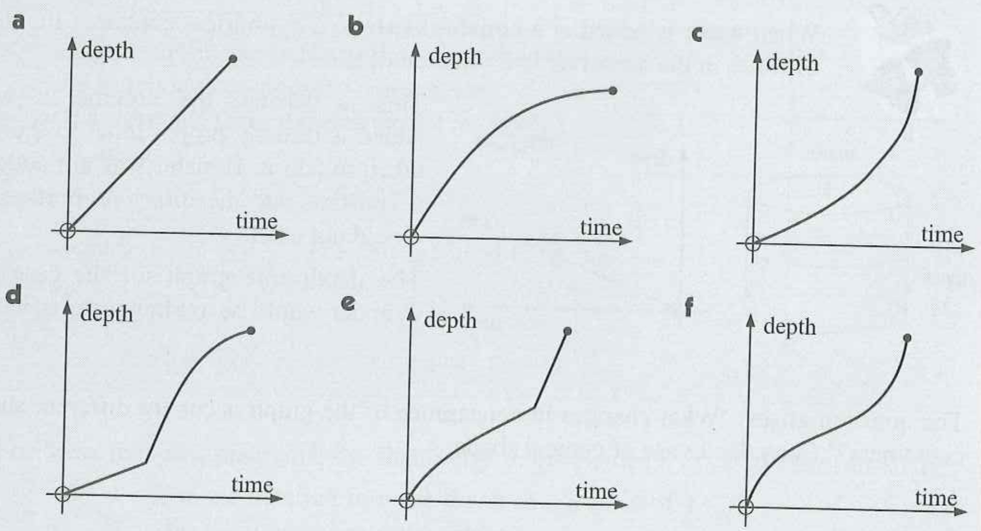
1 For each of the following containers, draw a 'depth v time' graph as water is added:



2 Use the water filling demonstration to check your answers to question **1**.

3 Write a brief report on the connection between the shape of a vessel and the corresponding shape of its depth-time graph. You may wish to discuss this in parts. For example, first examine cylindrical containers, then conical, then other shapes. Slopes of curves must be included in your report.

4 Draw possible containers as in question **1** which have the following 'depth v time' graphs:

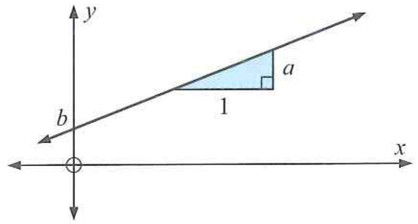


E LINEAR FUNCTIONS

A **linear function** is a function of the form $f(x) = ax + b$ or $f : x \mapsto ax + b$ where $a \neq 0$.

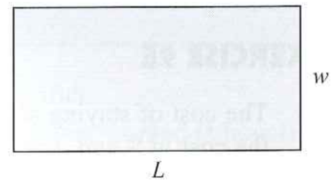
The *variables* are x and y whilst a and b are *constants*.

If we graph y against x we obtain a **straight line**, with *gradient* a and *y-intercept* b.



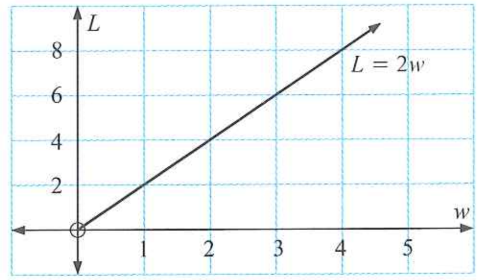
A GEOMETRICAL MODEL

Consider all rectangles with length twice their width. This information does not tell us how long or how wide the rectangle actually is.



However, it does give us a connection between these two variables. This is $L = 2w$ or using functional notation $L(w) = 2w$.

The graph is:



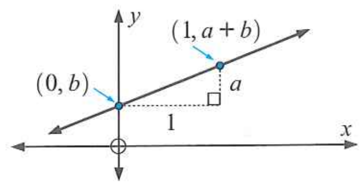
Notice in this case that the *domain* is $\{w: \ w > 0\}$ as sides of rectangles have positive lengths.

Clearly w and L are continuous variables and so the **full line** (not separate discrete points) is used to illustrate the relationship.

If a rectangle has width 5.23 m, then its length is $L(5.23)$
$= 2 \times 5.23$
$= 10.46$ m

KEY FEATURES OF LINEAR FUNCTION GRAPHS (REVIEW)

- The **x-intercept**: The x-value where the graph cuts the x-axis. Let $y = 0$ or $f(x) = 0$ and solve for x.
- The **y-intercept**: The y-value where the graph cuts the y-axis. Let $x = 0$, solve $f(0) = b$. b is the y-intercept.
- The **gradient**: As $f(1) = a + b$ then $(1, a+b)$ lies on the line. So, gradient $= \dfrac{y\text{-step}}{x\text{-step}} = a$.



Example 8

The cost of hiring a tennis court is given by the formula $C(h) = 5h + 8$ where C is the cost in \$ and h is the number of hours the court is hired for. Find the cost of hiring the tennis court for: **a** 4 hours **b** 10 hours.

The formula is $C(h) = 5h + 8$

a Substituting $h = 4$ we get

$$\begin{aligned} C(4) &= 5(4) + 3 \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

i.e., it costs \$28 for 4 hours.

b Substituting $h = 10$ we get

$$\begin{aligned} C(10) &= 5(10) + 8 \\ &= 50 + 8 \\ &= 58 \end{aligned}$$

i.e., it costs \$58 for 10 hours.

EXERCISE 9E

1 The cost of staying at a hotel is given by the formula, $C(d) = 50d + 20$ where C is the cost in \$ and d is the number of days a person stays. Find the cost of staying for:

a 3 days **b** 6 days **c** 2 weeks

2 The thermometer in a kitchen oven was designed using the Celsius (T_c) scale. However many recipe books give the required temperatures on the Fahrenheit (F) scale. The formula which links the two temperature scales is:

$$T_c(F) = \tfrac{5}{9}(F - 32)$$

Convert the following Fahrenheit temperatures into Celsius:

a 212°F **b** 32°F **c** 104°F **d** 374°F

3 If the value of a car t years after purchase is given by $V(t) = 25\,000 - 3000t$ dollars:

a find $V(0)$ and state the meaning of $V(0)$

b find $V(3)$ and state the meaning of $V(3)$

c find t when $V(t) = 10\,000$ and explain what this represents.

4 Find $f(x) = ax + b$, a linear function, in which $f(2) = 7$ and $f(-1) = -5$.

Example 9

Ace taxi services charge \$3.30 for stopping to pick up a passenger and then \$1.75 per km travelled thereafter.

a Copy and complete:

Distance (km)	0	2	4	6	8	10
Cost (\$$C$)						

b Graph C against d.

c Find the rule connecting the variables.

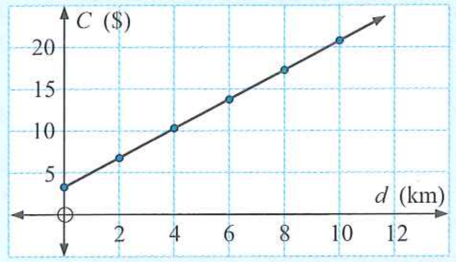
d Find the cost of a 9.4 km trip.

a

Distance (d km)	0	2	4	6	8	10
Cost (\$$C$)	3.30	6.80	10.30	13.80	17.30	20.80

adding 2 × \$1.75 each time

b



c 'y-intercept' is 3.30 and

gradient $= \dfrac{20.80 - 17.30}{10 - 8} = 1.75$

$\therefore \quad C(d) = 1.75d + 3.3$

d

$$\begin{aligned} C(9.4) &= 1.75 \times 9.4 + 3.3 \\ &= 19.75 \end{aligned}$$

$\therefore$ cost is \$19.75

5 An electrician charges \$60 for calling and \$45 per hour thereafter.

a From a table of values, plot the charge (\$$C$) against the hours worked (t hours) for $t = 0$, 1, 2, 3, 4 and 5.

b Use your graph to determine the cost function C, in terms of t.

c Use the cost function to determine the electrician's total cost for a job lasting $6\frac{1}{2}$ hours. Use your graph to check your answer.

6 A rainwater tank contains 265 litres. The tap is left on and 11 litres escape per minute.

a Construct a table of values of volume (V litres) left in the tank after time t (minutes) for $t = 0$, 1, 2, 3, 4 and 5.

b Use your table to graph V against t.

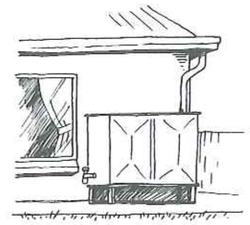
c Use your graph to determine the rule connecting V and t.

d Use your rule to determine:

i how much water is left in the tank after 15 minutes

ii the time taken for the tank to empty.

e Use your graph to check your answers to **d i** and **d ii**.



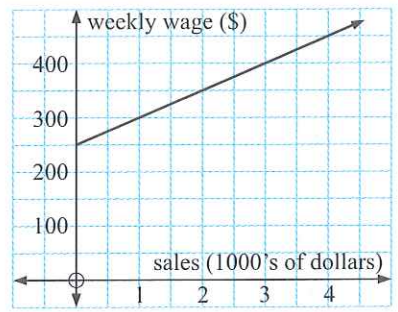
7 The cost of running a truck is \$158 plus \$365 per one thousand kilometres thereafter.

a Without graphing, determine the cost (\$$C$) in terms of the number of thousands of kilometres (n).

b Find the cost of running the truck a distance of 3750 km.

c How far could the truck travel if \$5000 was available?

8 A salesperson's wage is determined from the graph alongside.



a Determine the weekly wage (\$$W$) in terms of the sales (\$$s$ thousand dollars).

b Find the weekly wage when sales were \$33 500.

c Determine the sales necessary for a weekly wage of \$830.

Sometimes linear models are appropriate in manufacturing and other business situations.

Example 10

Nonchar electric toasters can 'set up' production of a new line of toaster for \$3000 and every 100 toasters produced thereafter will cost \$1000 to make. The toasters are then sold to a distributor for \$25 each and the distributor places an order for 1500 of them.

a Determine the cost of production function $C(n)$ where n is the number of toasters manufactured.

b Determine the revenue function $R(n)$.

c Graph $C(n)$ and $R(n)$ on the same set of axes.

d How many toasters need to be produced in order to 'break even'?

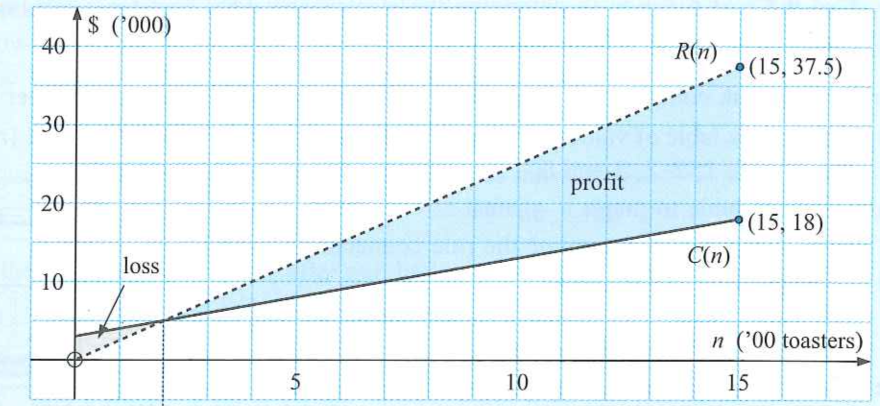
e Calculate the profit made on producing: **i** 800 toasters **ii** 1500 toasters.

a Each toaster costs \$10 to produce and the fixed cost is \$3000.

$\therefore\ C(n) = 10n + 3000$ dollars {gradient = cost/item = \$10}

b $R(n) = n \times \$25$ i.e., $R(n) = 25n$ dollars

c



d We see that to the left of the point of intersection $C(n) > R(n)$ so a loss is made.

So 'break even' is where $C(n) = R(n)$

$$\begin{aligned} \text{i.e., } 10n + 3000 &= 25n \\ \therefore\ 3000 &= 15n \\ \therefore\ n &= 200 \end{aligned}$$

200 toasters must be produced to 'break even'.

e Profit $= R(n) - C(n)$

$\therefore\ P(n) = 25n - (10n + 3000)$

$= 15n - 3000$

i $P(800) = 15 \times 800 - 3000$
$= \$9000$

ii $P(1500) = 15 \times 1500 - 3000$
$= \$19\,500$

9 Self adhesive label packs are produced with a cost function, $C(n) = 3n + 20$ dollars and a revenue function, $R(n) = 5n + 10$ dollars and n is the number of packs produced.

a Graph each function on the same set of axes, clearly labelling each graph.

b Determine the 'break even' production number. Check your answer algebraically.

c For what values of n is a profit made?

d How many self adhesive label packs should be produced to make a profit of \$100?

10 Two way adaptors sell for \$7 each. The adaptors cost \$2.50 each to make with fixed costs of \$300 per day regardless of the number made.

a Find revenue and cost functions in terms of the number manufactured, n.

b On the same set of axes graph the revenue and cost functions.

c Determine the 'break even' level of production and check your answer algebraically.

d What level of production will guarantee a profit of \$1000 per day?

11 A new novel is being printed. The production costs are \$6000 (fixed) and \$3250 per thousand books thereafter. The books are to sell at \$9.50 each with an unlimited market.

a Determine cost and revenue functions for the production of the novel.

b On the same set of axes graph the cost and revenue functions.

c How many books must be sold in order to 'break even'? Check your answer algebraically.

d What level of production will produce a \$10 000 profit?

12 Waverley Manufacturing produces carburettors for motor vehicles. At the beginning of each week it costs \$2100 to start up the factory. Each carburettor costs \$13.20 in materials and \$14.80 in labour to produce. Waverley is able to sell the carburettors to the motor vehicle manufacturers at \$70 each.

a Determine Waverley's cost and revenue functions in terms of the number (n) manufactured per week.

b Draw graphs of both of the functions in **a** on the same axes and use your graph to find the "break even" point.

c Find an expression for the profit function and check your answer for the "break even" value of n.

d Use your profit function to find:

i the weekly profit for producing 125 carburettors

ii the number of carburettors required to make a profit of at least \$1300.

USING TECHNOLOGY

Use a **graphics calculator** or **graphing package** to check some answers to the previous exercises.

GRAPHING PACKAGE

c $x^3-8x^2+19x-12$ **d** $2x^3+x^2-5x+2$
e $3x^3+14x^2+17x+6$ **f** $4x^3+16x^2-x-4$
g $-3x^3+7x^2-4$ **h** $-3x^3+10x^2-x-6$

5 **a** 4 **b** 6 **c** 6 **d** 9 **e** 8 **f** 12 **g** 8 **h** 12

EXERCISE 8C.1

1 **a** $x(3x+5)$ **b** $x(2x-7)$ **c** $3x(x+2)$ **d** $4x(x-2)$ **e** $x(2x-9)$ **f** $3x(x+5)$ **g** $4x(1+2x)$ **h** $5x(1-2x)$ **i** $4x(3-x)$

2 **a** $(x+2)(x-3)$ **b** $(x-1)(x-4)$ **c** $(x+1)(x+3)$ **d** $(x-2)(x+1)$ **e** $(x+3)(x+4)$ **f** $(x+4)(x+6)$ **g** $(x-3)(x-4)$ **h** $(x+4)(x+2)$ **i** $(x-4)(x-9)$

EXERCISE 8C.2

1 **a** $(x+2)(x-2)$ **b** $(2+x)(2-x)$ **c** $(x+9)(x-9)$ **d** $(5+x)(5-x)$ **e** $(2x+1)(2x-1)$ **f** $(3x+4)(3x-4)$ **g** $(2x+3)(2x-3)$ **h** $(6+7x)(6-7x)$

2 **a** $3(x+3)(x-3)$ **b** $-2(x+2)(x-2)$ **c** $3(x+5)(x-5)$ **d** $-5(x+1)(x-1)$ **e** $2(2x+3)(2x-3)$ **f** $-3(3x+5)(3x-5)$

3 **a** $(x+\sqrt{3})(x-\sqrt{3})$ **b** no linear factors **c** $(x+\sqrt{15})(x-\sqrt{15})$ **d** $3(x+\sqrt{5})(x-\sqrt{5})$ **e** $(x+1+\sqrt{6})(x+1-\sqrt{6})$ **f** no linear factors **g** $(x-2+\sqrt{7})(x-2-\sqrt{7})$ **h** $(x+3+\sqrt{17})(x+3-\sqrt{17})$ **i** no linear factors

4 **a** $(x+3)(x-1)$ **b** $4(x+2)(x-1)$ **c** $(x-5)(x+3)$ **d** $3(x+1)(3-x)$ **e** $(3x+2)(x-2)$ **f** $(2x+3)(4x-3)$ **g** $(3x-1)(x+3)$ **h** $8x(x-1)$ **i** $-3(4x+3)$

EXERCISE 8C.3

1 **a** $(x+3)^2$ **b** $(x+4)^2$ **c** $(x-3)^2$ **d** $(x-4)^2$ **e** $(x+1)^2$ **f** $(x-5)^2$ **g** $(y+9)^2$ **h** $(m-10)^2$ **i** $(t+6)^2$

2 **a** $(3x+1)^2$ **b** $(2x-1)^2$ **c** $(3x+2)^2$ **d** $(5x-1)^2$ **e** $(4x+3)^2$ **f** $(5x-2)^2$ **g** $(x-1)^2$ **h** $2(x+2)^2$ **i** $3(x+5)^2$

EXERCISE 8C.4

1 **a** $(x+2)(x+1)$ **b** $(x+3)(x+2)$ **c** $(x-3)(x+2)$ **d** $(x+5)(x-2)$ **e** $(x+7)(x-3)$ **f** $(x+4)^2$ **g** $(x-7)^2$ **h** $(x+7)(x-4)$ **i** $(x+5)(x+2)$ **j** $(x-8)(x-3)$ **k** $(x+11)(x+4)$ **l** $(x+7)(x-6)$ **m** $(x-8)(x+7)$ **n** $(x-9)^2$ **o** $(x-8)(x+4)$

2 **a** $2(x-4)(x+1)$ **b** $3(x+4)(x-1)$ **c** $5(x+3)(x-1)$ **d** $4(x+5)(x-4)$ **e** $2(x-5)(x+3)$ **f** $3(x+7)(x-3)$ **g** $2(x-5)(x+4)$ **h** $3(x-2)^2$ **i** $7(x+4)(x-1)$ **j** $5(x+5)(x-2)$ **k** $2(x+7)(x-3)$ **l** $(x-8)(x+4)$

EXERCISE 8C.5

1 **a** $3x(x+3)$ **b** $(2x+1)(2x-1)$ **c** $5(x+\sqrt{3})(x-\sqrt{3})$ **d** $x(3-5x)$ **e** $(x+8)(x-5)$ **f** $2(x+4)(x-4)$ **g** does not factorise **h** $(x+5)^2$ **i** $(x-3)(x+2)$ **j** $(x-13)(x-3)$ **k** $(x-12)(x+5)$ **l** $(x-4)(x+2)$ **m** $(x+5)(x+6)$ **n** $(x+8)(x-2)$ **o** $(x-8)(x+3)$ **p** $3(x+6)(x-4)$ **q** $4(x-5)(x+3)$ **r** $3(x-11)(x-3)$

EXERCISE 8D

1 **a** $(2x+3)(x+1)$ **b** $(2x+5)(x+1)$ **c** $(7x+2)(x+1)$ **d** $(3x+4)(x+1)$ **e** $(3x+1)(x+4)$ **f** $(3x+2)(x+2)$ **g** $(4x+1)(2x+3)$ **h** $(7x+1)(3x+2)$ **i** $(3x+1)(2x+1)$ **j** $(6x+1)(x+3)$ **k** $(5x+1)(2x+3)$ **l** $(7x+1)(2x+5)$

2 **a** $(2x+1)(x-5)$ **b** $(3x-1)(x+2)$ **c** $(3x+1)(x-2)$ **d** $(2x-1)(x+2)$ **e** $(2x+5)(x-1)$ **f** $(5x+1)(x-3)$ **g** $(5x-3)(x-1)$ **h** $(11x+2)(x-1)$ **i** $(3x+2)(x-3)$ **j** $(2x+3)(x-3)$ **k** $(3x-2)(x-5)$ **l** $(5x+2)(x-3)$ **m** $(3x-2)(x+4)$ **n** $(2x-1)(x+9)$ **o** $(2x-3)(x+6)$ **p** $(2x-3)(x+7)$ **q** $(5x+2)(3x-1)$ **r** $(21x+1)(x-3)$ **s** $(3x-2)^2$ **t** $(4x-5)(3x+8)$ **u** $(8x-3)(2x+5)$

EXERCISE 8E.1

1 **a** $x=\pm 4$ **b** $x=\pm 3$ **c** $x=\pm 3$ **d** $x=\pm\sqrt{6}$ **e** no solution **f** $x=0$ **g** $x=\pm 3$ **h** no solution **i** $x=\pm\sqrt{3}$

2 **a** $x=5$ or -1 **b** $x=1$ or -9 **c** no solution **d** $x=4\pm\sqrt{2}$ **e** no solution **f** $x=-2$ **g** $x=-2\frac{1}{2}$ **h** $x=0$ or $\frac{4}{3}$ **i** $x=\dfrac{1\pm\sqrt{24}}{2}$

EXERCISE 8E.2

1 **a** $x=0$ or -3 **b** $x=0$ or 5 **c** $x=1$ or 3 **d** $x=0$ or 2 **e** $x=0$ or $-\frac{1}{2}$ **f** $x=-2$ or $\frac{1}{2}$ **g** $x=-\frac{1}{2}$ or $\frac{1}{2}$ **h** $x=-2$ or 7 **i** $x=5$ or $-\frac{2}{3}$ **j** $x=0$ **k** $x=3$ **l** $x=\frac{1}{2}$

EXERCISE 8E.3

1 **a** $x=0$ or -2 **b** $x=0$ or $-\frac{5}{2}$ **c** $x=0$ or $\frac{3}{4}$ **d** $x=0$ or $\frac{5}{4}$ **e** $x=0$ or 3 **f** $x=0$ or $\frac{1}{2}$

2 **a** $x=-7$ or -2 **b** $x=-5$ or -6 **c** $x=-5$ or 3 **d** $x=-4$ or 3 **e** $x=3$ or 2 **f** $x=2$ **g** $x=3$ or -2 **h** $x=12$ or -5 **i** $x=10$ or -7 **j** $x=-5$ or 2 **k** $x=3$ or 4 **l** $x=12$ or -3

3 **a** $x=\frac{1}{2}$ or 2 **b** $x=-3$ or $\frac{1}{3}$ **c** $x=-4$ or $-\frac{5}{3}$ **d** $x=\frac{1}{2}$ or -3 **e** $x=\frac{1}{2}$ or 5 **f** $x=-1$ or $-\frac{5}{2}$ **g** $x=-\frac{1}{3}$ or -4 **h** $x=-\frac{2}{5}$ or 3 **i** $x=\frac{1}{2}$ or -9 **j** $x=1$ or $-\frac{5}{2}$ **k** $x=\frac{4}{3}$ or -2 **l** $x=\frac{3}{2}$ or -6

4 **a** $x=\frac{1}{3}$ or $-\frac{5}{2}$ **b** $x=\frac{2}{3}$ or $-\frac{1}{2}$ **c** $x=-\frac{1}{2}$ or $-\frac{1}{3}$ **d** $x=-\frac{1}{21}$ or 3 **e** $x=\frac{2}{5}$ or $-\frac{1}{2}$ **f** $x=-\frac{3}{10}$ or 1

5 **a** $x=-4$ or -3 **b** $x=-3$ or 1 **c** $x=\pm 3$ **d** $x=-1$ or $\frac{2}{3}$ **e** $x=-\frac{1}{2}$ **f** $x=\frac{5}{2}$ or 4

6 **a** $x=\pm\sqrt{6}$ **b** $x=\pm\sqrt{8}$ **c** $x=\pm\sqrt{10}$ **d** $x=4$ or -3 **e** $x=-1$ or -5 **f** $x=2$ or -1 **g** $x=\frac{1}{2}$ or -1 **h** $x=1$ or $-\frac{1}{3}$ **i** $x=-1$ or 4

EXERCISE 8F

1 **a** **i** 1 **ii** $(x+1)^2=6$ **b** **i** 1 **ii** $(x-1)^2=-6$ **c** **i** 9 **ii** $(x+3)^2=11$ **d** **i** 9 **ii** $(x-3)^2=6$ **e** **i** 25 **ii** $(x+5)^2=26$ **f** **i** 16 **ii** $(x-4)^2=21$ **g** **i** 36 **ii** $(x+6)^2=49$ **h** **i** $\frac{25}{4}$ **ii** $(x+\frac{5}{2})^2=4\frac{1}{4}$ **i** **i** $\frac{49}{4}$ **ii** $(x-\frac{7}{2})^2=16\frac{1}{4}$

2 **a** $x=2\pm\sqrt{3}$ **b** $x=1\pm\sqrt{3}$ **c** $x=2\pm\sqrt{7}$ **d** $x=-1\pm\sqrt{2}$ **e** no solution **f** $x=-2\pm\sqrt{3}$

g $x=-3\pm\sqrt{6}$ **h** no solution **i** $x=-4\pm\sqrt{2}$

3 **a** $x=-1$ or -2 **b** $x=2\pm\sqrt{12}$ **c** $x=2$ or 3 **d** $x=\dfrac{-1\pm\sqrt{5}}{2}$ **e** $x=\dfrac{-3\pm\sqrt{13}}{2}$ **f** $x=\dfrac{-5\pm\sqrt{33}}{2}$

4 **a** -0.551 or -5.45 **b** no solutions **c** -2.59 or -5.41 **d** 0.225 or -2.22 **e** 3.29 or 0.709 **f** 1.63 or 0.368

EXERCISE 8G

1 8 or -9 **2** 8 or -12 **3** 8 or -3 **4** 15 and -6

5 -2 and 3 or 2 and -3 **6** width is 7 cm

7 altitude is 9 cm **8** 8 m × 12 m

9 Either 5 m from wall and 14 m wide *or* 7 m from wall and 10 m wide.

10 **a** $x=5$ **b** $x=6$ **11** 8 cm, 15 cm, 17 cm

12 15 rows **13** BC is 5 cm or 16 cm **14** 3 cm × 3 cm

15 **c** $1\frac{1}{2}$ metres wide

REVIEW SET 8A

1 **a** $2x-10$ **b** $7-7x$

2 **a** $3x^2+4x-4$ **b** $4x^2+4x+1$ **c** $9x^2-1$

3 **a** $(x-7)(x+3)$ **b** $(2x+5)(2x-5)$ **c** $3(x-6)(x+4)$ **d** $(3x+2)(2x-1)$

4 **a** $x=3$ or 8 **b** $x=-\frac{2}{5}$ or $\frac{3}{2}$

5 $(x+3)^2=-2$ has no solution **6** 13 cm × 20 cm

7 **a** $-x^2-8x-4$ **b** $3x^3+4x^2+17x-14$

REVIEW SET 8B

1 **a** $-x$ **b** $2x^2-5x$

2 **a** $2x^2+x-15$ **b** $9x^2-24x+16$ **c** $9x^2-4$

3 **a** $(x-11)(x+3)$ **b** $(x+8)(x-4)$ **c** $(x-5)^2$

4 **a** $x=-8$ or 3 **b** $x=\pm 3$ **c** $x=-\frac{3}{4}$ or $\frac{1}{2}$

5 $x=1\pm\sqrt{101}$ **6** 9 cm × 12 cm **7** 5 or 2

8 **a** $5x^2-5x-5$ **b** $2x^3-3x^2+4x+3$

REVIEW SET 8C

1 **a** $-14x+35$ **b** $7x-18$

2 **a** $5x^2+14x-3$ **b** $9x^2-6x+1$ **c** $1-25x^2$

3 **a** $(x-9)(x+2)$ **b** $(2x-7)(2x+7)$ **c** $2(x-10)(x+3)$ **d** $(5x+2)(x+2)$

4 **a** $x=2$ or 3 **b** $x=4$ **c** $x=-\frac{7}{3}$ or 3

5 $x=7\pm\sqrt{42}$ **6** The number is $2+\sqrt{3}$ or $2-\sqrt{3}$.

7 8 cm, 15 cm, 17 cm

8 **a** $-x^2-3x-15$ **b** $2x^3+3x^2+x+15$

EXERCISE 9A

1 a, d, e **2** a, b, c, e, h **3** No, e.g., $x=1$

4 $y=\pm\sqrt{9-x^2}$, i.e., a vertical line cuts it more than once.

EXERCISE 9B

1 **a** Domain $\{x:\ x\geqslant -1\}$, Range $\{y:\ y\leqslant 3\}$
b Domain $\{x:\ -1<x\leqslant 5\}$, Range $\{y:\ 1<y\leqslant 3\}$
c Domain $\{x:\ x\neq 2\}$, Range $\{y:\ y\neq -1\}$
d Domain $\{x:\ x\in R\}$, Range $\{y:\ 0<y\leqslant 2\}$
e Domain $\{x:\ x\in R\}$, Range $\{y:\ y\geqslant -1\}$
f Domain $\{x:\ x\in R\}$, Range $\{y:\ y\leqslant \frac{25}{4}\}$
g Domain $\{x:\ x\geqslant -4\}$, Range $\{y:\ y\geqslant -3\}$
h Domain $\{x:\ x\in R\}$, Range $\{y:\ y>-2\}$
i Domain $\{x:\ x\neq \pm 2\}$, Range $\{y:\ y\leqslant -1$ or $y>0\}$

2 **a** Domain $\{x:\ x\geqslant 0\}$, Range $\{y:\ y\geqslant 0\}$
b Domain $\{x:\ x\neq 0\}$, Range $\{y:\ y>0\}$
c Domain $\{x:\ x\leqslant 4\}$, Range $\{y:\ y\geqslant 0\}$
d Domain $\{x:\ x\in R\}$, Range $\{y:\ y\geqslant -2\frac{1}{4}\}$
e Domain $\{x:\ x\in R\}$, Range $\{y:\ y\leqslant 2\frac{1}{12}\}$
f Domain $\{x:\ x\neq 0\}$, Range $\{y:\ y\leqslant -2$ or $y\geqslant 2\}$
g Domain $\{x:\ x\neq 2\}$, Range $\{y:\ y\neq 1\}$
h Domain $\{x:\ x\in R\}$, Range $\{y:\ y\in R\}$
i Domain $\{x: x\neq -1$ or $2\}$, Range $\{y: y\leqslant \frac{1}{3}$ or $y\geqslant 3\}$
j Domain $\{x:\ x\neq 0\}$, Range $\{y: y\geqslant 2\}$
k Domain $\{x:\ x\neq 0\}$, Range $\{y:\ y\leqslant -2$ or $y\geqslant 2\}$
l Domain $\{x:\ x\in R\}$, Range $\{y:\ y\geqslant -8\}$

EXERCISE 9C

1 **a** 2 **b** 8 **c** -1 **d** -13 **e** 1

2 **a** -3 **b** 3 **c** 3 **d** -3 **e** $7\frac{1}{2}$

3 **a** 2 **b** 2 **c** -16 **d** -68 **e** $\frac{17}{4}$

4 **a** $7-3a$ **b** $7+3a$ **c** $-3a-2$ **d** $10-3b$ **e** $1-3x$

5 **a** $2x^2+19x+43$ **b** $2x^2-11x+13$ **c** $2x^2-3x-1$ **d** $2x^4+3x^2-1$ **e** $2x^4-x^2-2$

6 **a** **i** $-\frac{7}{2}$ **ii** $-\frac{3}{4}$ **iii** $-\frac{4}{9}$ **b** $x=4$ **c** $\dfrac{2x+7}{x-2}$ **d** $x=\frac{9}{5}$

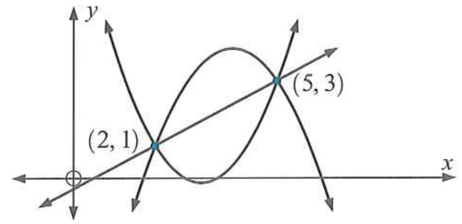
7 f is the function which converts x into $f(x)$ whereas $f(x)$ is the value of the function at any value of x.

8 $f(a)f(b)=2^a2^b=2^{a+b}$ {index law} and $f(a+b)=2^{a+b}$

9 **a** $x+3$ **b** $4+h$

10 **a** \$6210, value after 4 years **b** $t=4.5$, the time for the photocopier to reach a value of \$5780. **c** \$9650

11

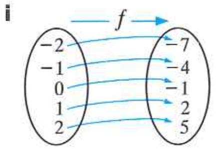


12 $f(x)=-2x+5$ **13** $a=3,\ b=-2$

14 $a=3,\ b=-1,\ c=-4,\ T(x)=3x^2-x-4$

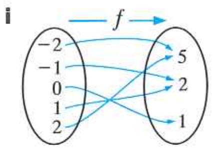
EXERCISE 9D

1 **a** **i**



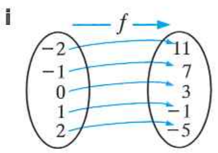
ii domain is $\{-2, -1, 0, 1, 2\}$
iii range is $\{-7, -4, -1, 2, 5\}$

b **i**

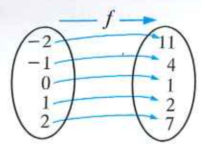


ii domain is $\{-2, -1, 0, 1, 2\}$
iii range is $\{1, 2, 5\}$

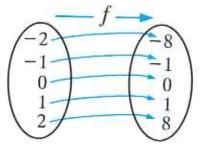
c **i**



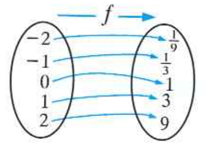
ii domain is $\{-2, -1, 0, 1, 2\}$
iii range is $\{-5, -1, 3, 7, 11\}$

d **i** 

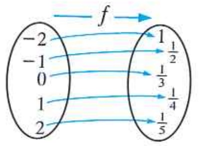
ii domain is $\{-2, -1, 0, 1, 2\}$
iii range is $\{1, 2, 4, 7, 11\}$

e **i** 

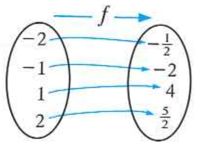
ii domain is $\{-2, -1, 0, 1, 2\}$
iii range is $\{-8, -1, 0, 1, 8\}$

f **i** 

ii domain is $\{-2, -1, 0, 1, 2\}$
iii range is $\{\frac{1}{9}, \frac{1}{3}, 1, 3, 9\}$

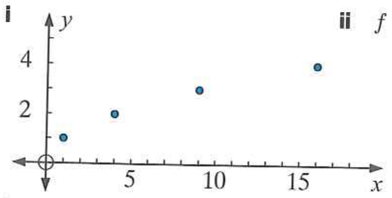
g **i** 

ii domain is $\{-2, -1, 0, 1, 2\}$
iii range is $\{\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1\}$

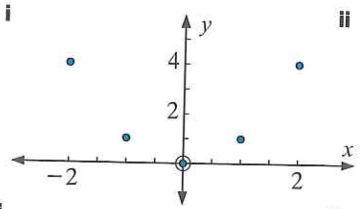
h **i** 

ii domain is $\{-2, -1, 1, 2\}$
iii range is $\{-2, -\frac{1}{2}, \frac{5}{2}, 4\}$

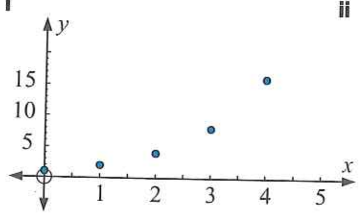
2 **a** **i** domain is $\{-2, -1, 0, 1, 2\}$
ii range is $\{1, 3, 5, 7, 9\}$ **iii** $f : x \mapsto 2x + 5$
b **i** domain is $\{0, 1, 2, 3, 4\}$
ii range is $\{1, 2, 3, 4, 5\}$ **iii** $f : x \mapsto 5 - x$
c **i** domain is $\{-3, 0, 3, 6, 9\}$
ii range is $\{-23, -11, 1, 13, 25\}$ **iii** $f : x \mapsto 4x - 11$
d **i** domain is $\{-3, 3, 6, 9, 15\}$
ii range is $\{-23, -11, -5, 1, 13\}$
iii $f : x \mapsto -2x + 7$

3 **a** **i** 

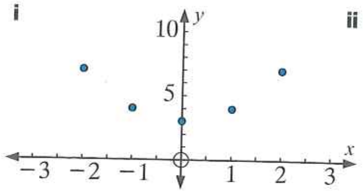
ii $f : x \mapsto \sqrt{x}$

b **i** 

ii $f : x \mapsto x^2$

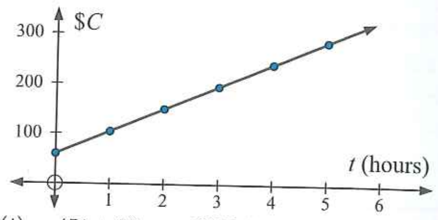
c **i** 

ii $f : x \mapsto 2^x$

d **i** 

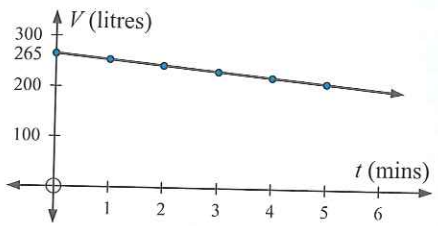
ii $f : x \mapsto x^2 + 3$

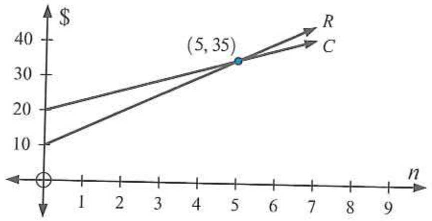
EXERCISE 9E

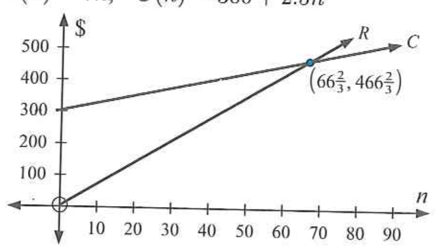
1 **a** \$170 **b** \$320 **c** \$720
2 **a** 100°C **b** 0°C **c** 40°C **d** 190°C
3 **a** $V(0) = \$25\,000$, $V(0)$ is the initial value of the car
b $V(3) = \$16\,000$, $V(3)$ is the value of the car after 3 years
c $t = 5$, where 5 years is the time taken for the value of the car to decrease to \$10 000
4 $f(x) = 4x - 1$
5 **a** 

b $C(t) = 45t + 60$ **c** \$352.50
6 **a**

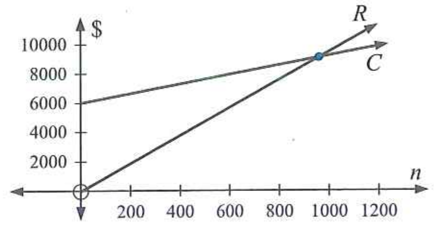
t	0	1	2	3	4	5
V	265	254	243	232	221	210

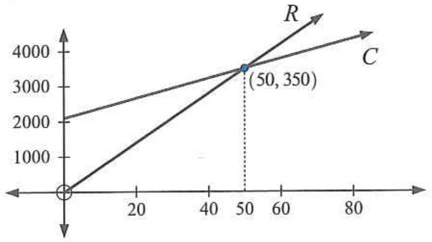
b 

c $V(t) = 265 - 11t$ **d** **i** 100 litres **ii** $\doteqdot 24.1$ min
7 **a** $C(n) = 158 + 365n$ **b** \$1526.75 **c** $\doteqdot 13\,266$ km
8 **a** $W(s) = 250 + 50s$ **b** \$1925 **c** \$11 600
9 **a** 

b $n = 5$ **c** $n > 5$ **d** 55
10 **a** $R(n) = 7n$, $C(n) = 300 + 2.5n$
b 

c 67 **d** 289

11 **a** $C(n) = 6000 + 3.25n$, $R(n) = 9.5n$
b 

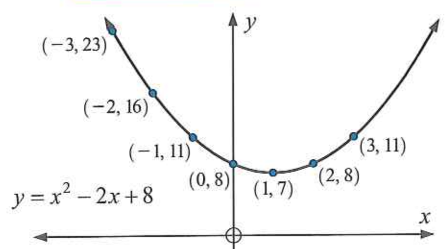
c 960 **d** 2560
12 **a** $C(n) = 2100 + 28n$, $R(n) = 70n$
b 

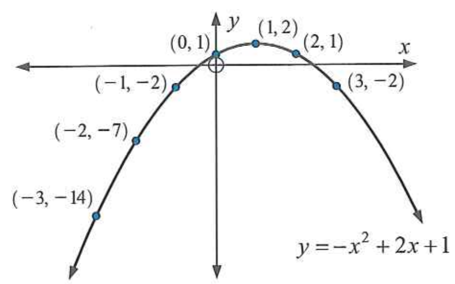
50 carburettors required to break even
c $P(n) = 42n - 2100$
d **i** \$3150 **ii** at least 81 carburettors

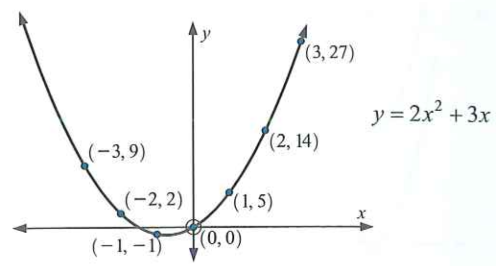
EXERCISE 9F

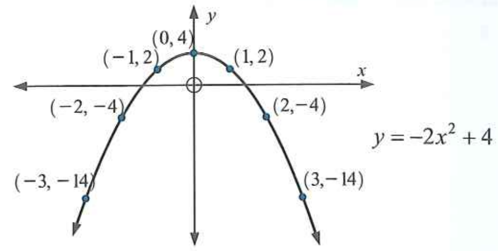
1 **a**, **c**, **d**, **e**
2 **a** $y = 20$ **b** $y = 27$ **c** $y = -4$ **d** $y = 37$
3 **a** 3 **b** -5 **c** 8
4 **a** no **b** no **c** no **d** no
5 **a** $x = -3$ **b** $x = -3$ or -2 **c** $x = 1$ or 4 **d** no solution
6 **a** $x = 0$ or $\frac{2}{3}$ **b** $x = 3$ or -2 **c** $x = \frac{1}{2}$ or -7 **d** $x = 3$
7 **a** **i** 25 m **ii** 25 m **iii** 45 m
b **i** 2 secs and 4 secs **ii** 0 secs and 6 secs
c once going up and once coming down
8 **a** **i** $-\$30$ **ii** \$105 **b** 6 or 58 cakes

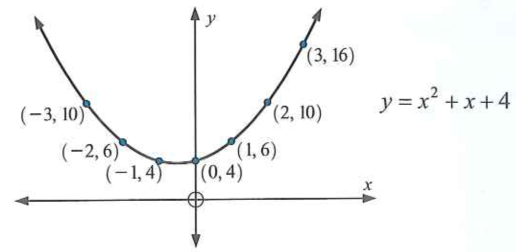
EXERCISE 9G.1

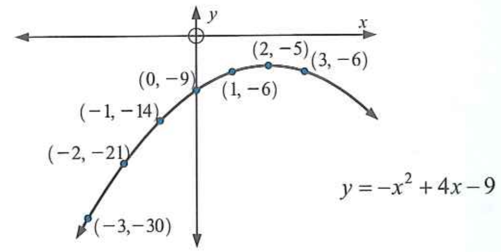
1 **a** 

b 

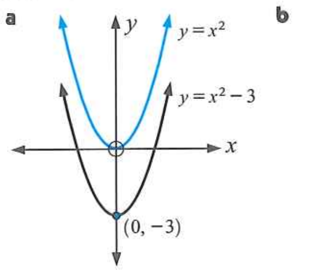
c 

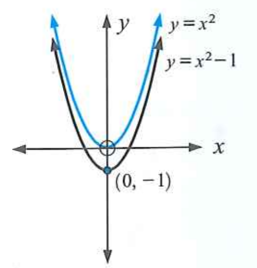
d 

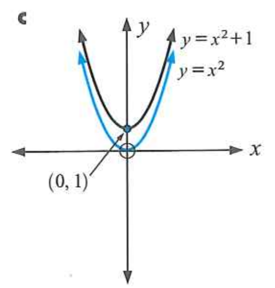
e 

f 

EXERCISE 9G.2

1 **a** 

b 

c 

d